9. RANDOM VARIABLE AND DISTRIBUTIONS

Quick Review

1. Let S be a sample space of a random experiment. A real valued function $X : S \rightarrow R$ is called a ***random variable.***

2. Let X be a random variable on a sample space S. If $x \in R$ then we use the following symbols to denote some events in S.

   i) $\{a \in S : X(a) = x\} = (X = x)$

   ii) $\{a \in S : X(a) < x\} = (X < x)$

   iii) $\{a \in S : X(a) \leq x\} = (X \leq x)$

   iv) $\{a \in S : X(a) > x\} = (X > x)$

   v) $\{a \in S : X(a) \geq x\} = (X \geq x)$

3. Let S be a sample space and $X : S \rightarrow R$ be a random variable. The function $F : R \rightarrow R$ defined by $F(x) = P(X \leq x)$, is called ***probability distribution function*** of the random variable X.

4. A set A is said to be ***countable*** if there exists a bijection (one one onto function) from A onto a subset of N.

5. If A is countable set, then A can be represented as $A = \{x_1, x_2, x_3, \ldots\}$.

6. Let S be a sample space. A random variable
   $X : S \rightarrow R$ is said to be ***discrete*** or ***discontinuous*** if the range of X is countable.

7. If $X : S \rightarrow R$ is a discrete random variable with range $\{x_1, x_2, x_3, \ldots\}$, then $\sum_{r=1}^{\infty} P(X = x_r) = 1$.

8. If $X : S \rightarrow R$ is a discrete random variable with range $\{x_1, x_2, x_3, \ldots\}$, then $\{P(X = x_r) : r = 1, 2, \ldots\}$ is called ***probability distribution*** of X.

9. Let $X : S \rightarrow R$ is a discrete random variable with range $\{x_1, x_2, x_3, \ldots\}$. If $\Sigma\, x_r\, P(X = x_r)$ exists, then $\Sigma\, x_r\, P(X = x_r)$ is called the ***mean*** of the random variable X. It is denoted by μ or $\bar{x}$. If $\Sigma\, (x_r - \mu)^2\, P(X = x_r)$ exists, then $\Sigma\, (x_r - \mu)^2\, P(X = x_r)$ is called ***variance*** of the random variable X. It is denoted by σ^2. The positive square root of the variance is called the ***standard deviation*** of the random variable X. It is denoted by σ.

10. Let $X : S \rightarrow R$ be a discrete random variable with range $\{x_1, x_2, x_3, \ldots\}$. If μ, σ^2 are the mean and variance of X, then $\sigma^2 + \mu^2 = \Sigma\, x_r^2\, P(X = x_r)$.

11. Let n be a positive integer and p be a real number such that $0 \leq p \leq 1$. A random variable X with range $\{0, 1, 2, \ldots, n\}$ is said to follow (or have) ***binomial distribution*** or ***Bernoulli distribution*** with parameters n and p, if $P(X = r) = {}^nC_r\, p^r q^{n-r}$ for $r = 0, 1, 2, \ldots n$, where $q = 1 - p$.

12. If the random variable X follows a binomial distribution with parameters n and p, then mean of X is np and the variance is npq, where $q = 1 - p$.

13. Let $\lambda > 0$ be a real number. A random variable X with range $\{0, 1, 2, \ldots\}$ is said to follow (have) ***Poisson distribution*** with parameter λ, if $P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$ for $r = 0, 1, 2, \ldots$

14. If a random variable X follows Poisson distribution with parameter λ, then mean of X is λ and variance of X is λ.